Chapter 5: Gases (Ch 10 in Jespersen, Ch 5 in Chang)

1A	f.																8A
н	2A	12										3A	4A	5A	6A	7A	He
Li	Be											в	с	N	0	F	Ne
Na	Mg	3B	4B	5B	6B	7B		- 8B -		1B	2B	AI	Si	Р	s	СІ	Ar
к	Ca	Sc	Ti	v	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Мо	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Hf	Та	w	Re	Os	Ir	Pt	Au	Hg	п	Pb	Bi	Ро	At	Rn
Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg							

Elements that exist as gases (at 25 °C and 1atm) are shown in blue.

Our atmosphere is: Nitrogen 78%, Oxygen 21%, Argon 0.9%, Carbon Dioxide 0.03%, Water vapor 0.0 to 4.0%.

Trace gases include neon, helium, krypton, and xenon.

Characteristics of Gases

- expand to fill container.
- can be *expanded* or *compressed*.
- form *homogeneous* mixtures (e.g. air).
- have much *lower densities* than liquids and solids.
- gases only occupy a *small* fraction of the total volume; they behave as if the other molecules were absent.
- they can exert a *pressure* (e.g. balloons).
- the pressure depends on the *amount* of confined gas.
- the pressure will rise and fall with *temperature*.

The fact that gases are readily compressible and immediately fill the space available to them suggests that molecules of gases are widely separated and in ceaseless chaotic motion.

Even though gases can vary tremendously in their *chemical* properties, essentially all gases obey the same set of *physical* properties.

These four inter-related physical properties are Pressure, Volume, Temperature and Amount.

Pressure (P)

Pressure is a force per unit area.

Force = **mass** (kg) x **acceleration** (ms⁻²).

The SI unit of **force** is the Newton, **N**.

The SI unit of **Pressure** is the **Pascal**, **Pa**.

$$P = \frac{F}{A} = \frac{kg \frac{m}{s^2}}{m^2} = \frac{N}{m^2} = Pa$$

Units of $F = N = kg \frac{m}{s^2}$

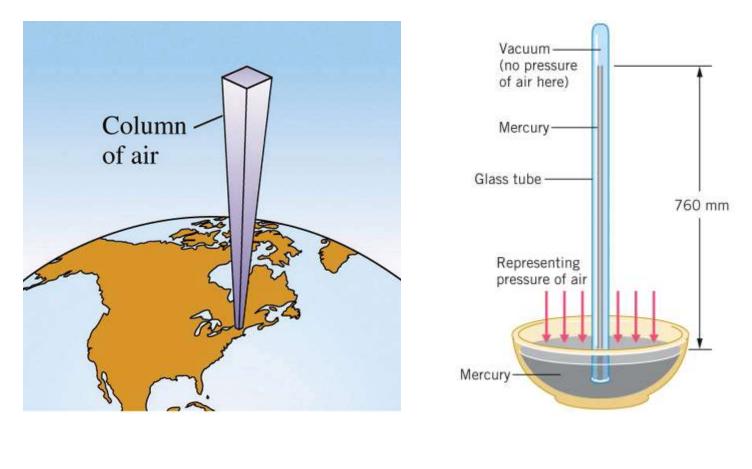
Pressures can also be expressed as *pressures that can support a certain sized column of mercury* (at a given elevation and temperature).

Standard atmospheric pressure (**1 atm**) is the pressure that supports a column of mercury 760 mm high at 0 °C at sea level.

1 atm = **760** mmHg = **760** torr = 1.01325×10^5 Pa = 101.325 kPa = 29.92 inHg = 14.696 psi

1 torr = 1 mmHg 1 bar = 10^5 Pa 1 bar \approx 1 atm.

Named after Torricelli who invented the barometer - a barometer is an instrument that measures atmospheric pressure.

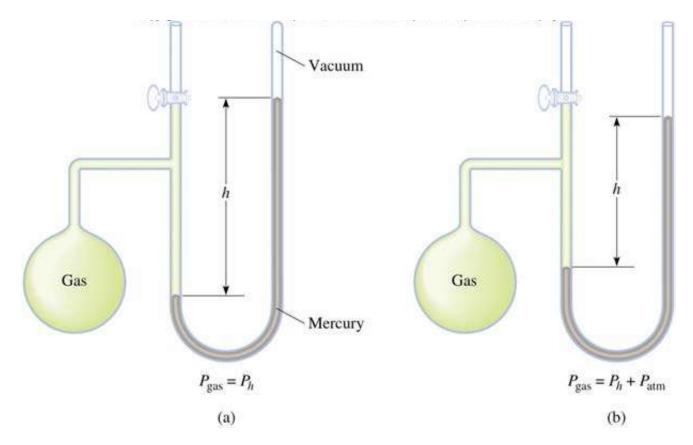


Mt Everest	29000 ft	253 mmHg	(low pressure)
Denver	5,690 ft	620 mmHg	
Dead Sea	1,407 ft below sea level	799 mmHg	(high pressure)

A manometer is an instrument that can measure pressure (of gases other than the atmosphere).

Either they are (a) closed tube (to measure pressure below atmospheric), or

(b) open tube (to measure pressure equal or greater than atmospheric pressure).



Mercury is used since it has such a *high density* (1.36 g/mL).

The Gas Laws

The gas laws were developed at the end of the 18th century, when scientists (through **experimental** observations) began to realize that relationships between the pressure, volume and temperature of a sample of gas could be obtained which would hold to a good approximation for all gases. Gases behave in a similar way over a wide variety of conditions because they all have *molecules* which are *widely spaced*.

Previously: these four inter-related physical properties are **Pressure**, **Volume**, **Temperature** and **Amount**.

Boyle's Law (the Pressure/Volume Relationship – the 1600's)

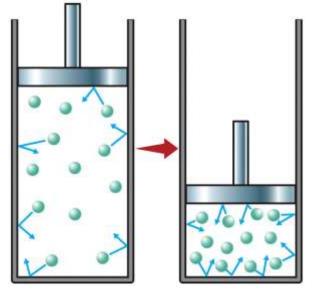
The pressure of a *fixed amount* of gas maintained at *constant temperature* is inversely proportional to the volume.

$$P \propto \frac{1}{V}$$

Also: $P = k \times \frac{1}{v}$

or

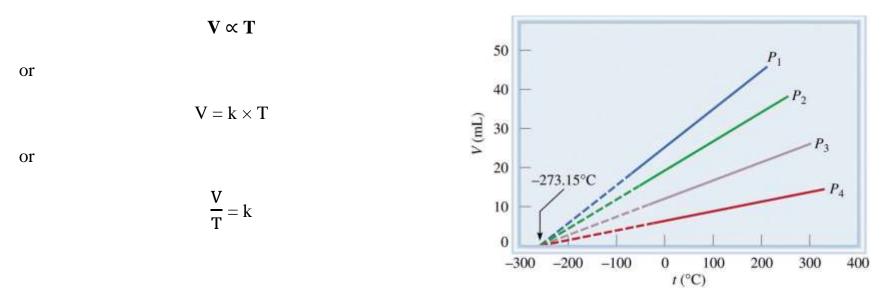
PV = k where k = constant.



Smaller Volume \Rightarrow *Larger* Pressure

Charles's Law (the Volume/Temperature Relationship – in the 1700's).

The volume of a *fixed amount* of gas maintained at *constant pressure* is directly proportional to its absolute temperature.



At any pressure, the plot of volume versus temperature gave a straight line.

By extending the line to zero volume, every line gave the same intercept on the temperature axis.

In 1848, Kelvin identified this temperature as absolute zero (set as 0 K on the absolute temperature scale), the theoretically lowest attainable temperature.

Gay-Lussac's Law (the Pressure/Temperature Relationship – in the 1800's).

The pressure of a *fixed amount* of gas maintained at *constant volume* is directly proportional to its absolute temperature.

or or $P = k \times T$ $\frac{P}{T} = k$ $\frac{P}{T} = k$ \frac{P}

Pressure increases as temperature increases.

Avogadro's Law (the Volume/Amount relationship - 1811)

At *constant pressure* and *temperature*, the volume of a gas is directly proportional to the number of moles of the gas present.

 $\mathbf{V} \propto \mathbf{n}$ (\mathbf{n} = amount of substance)

Or $V = k \times n$

So
$$\frac{\mathbf{V}}{\mathbf{n}} = \mathbf{k} = \mathbf{V}_{\mathbf{m}}$$
 or $\mathbf{V} = \mathbf{n}\mathbf{V}_{\mathbf{m}}$

Where V_m is the molar volume = $22.4 \frac{L}{mol}$ (more on this later)

(Equal volumes of gases at the same temperature and pressure contain equal numbers of molecules).

The Ideal Gas Equation

Boyle's law (constant n and T)
$$V \propto \frac{1}{P}$$
Charles's Law (constant n and P) $V \propto T$

Avogadro's Law (constant P and T) $V \propto n$

So if R is a (the Gas) constant: $V = R \frac{nT}{P}$

$\mathbf{PV} = \mathbf{nRT}$ Ideal Gas Law

An ideal gas is a *hypothetical* gas whose *pressure-volume-temperature behavior* can be completely accounted for by the **ideal gas equation**. (Ideal behavior is seen at lower pressures and higher temperatures).

For an ideal gas we pretend:

- Molecules of an ideal gas do **not** attract or repel one another.
- They have negligible volume compared to the container.

PV=nRT is the equation of state of a gas.

If we know three of the 4 variables, we can calculate the fourth.

We can define the state of a gas by defining three of these values. (See Ch 6 in my notes for more on "states").

The Gas Constant

If we apply **PV=nRT** at 0 °C (273.15 K), using 1 mol of ideal gas occupying 22.4 L (*established* by experiment), we get the value of R:

 $R = 0.08206 \ \frac{L \cdot atm}{mol \cdot K}$

Beware of units – R is sometimes expressed as
$$R = 8.314 \frac{J}{mol \cdot K}$$

Standard Temperature and Pressure (STP) is defined as:

STP $T = 273.15 \text{ K} = 0 \degree \text{C}$ P = 1.0000 atm

All of the other gas laws can be derived from the ideal gas law...

So we have: $\mathbf{PV} = \mathbf{nRT}$ where **R** is a **constant**.

In Chemistry we often deal with changes or modifications:

$$R = \frac{P_1 V_1}{n_1 T_1}$$
 (before the change) and $R = \frac{P_2 V_2}{n_2 T_2}$ (after the change)

Therefore:
$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$
 This is called the **Combined Gas Law.**

If the number of moles doesn't change (common for gas experiments, $n_1 = n_2$)

then:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

If other variables are constant we get these useful forms:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \qquad P_1 V_1 = P_2 V_2$$

Problem: An inflated balloon has a volume of 6.0 L at sea level (1.0 atm) and is allowed to ascend in altitude until the pressure is 0.45 atm. During the ascent the temperature of the gas falls from 22 °C to -21 °C.

Calculate the volume of the balloon at its final altitude.

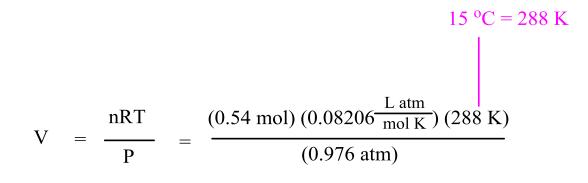
So we know:	$V_1 = 6.0 L$	V ₂ = ?
	$P_1 = 1.0 \text{ atm}$	$P_2 = 0.45 \text{ atm}$
	$T_1 = 22 \ ^{o}C$	$T_2 = -21 \ ^{o}C$
Use KELVINS in Gas problems!	$T_1 = 295 \text{ K}$	$T_2 = 252 \text{ K}$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \qquad \text{so} \qquad V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$$

$$= \frac{(1.0 \text{ atm}) (6.0 \text{ L}) (252 \text{ K})}{(295 \text{ K}) (0.45 \text{ atm})} = 11.4 \text{ L}$$
$$= 11. \text{ L}$$

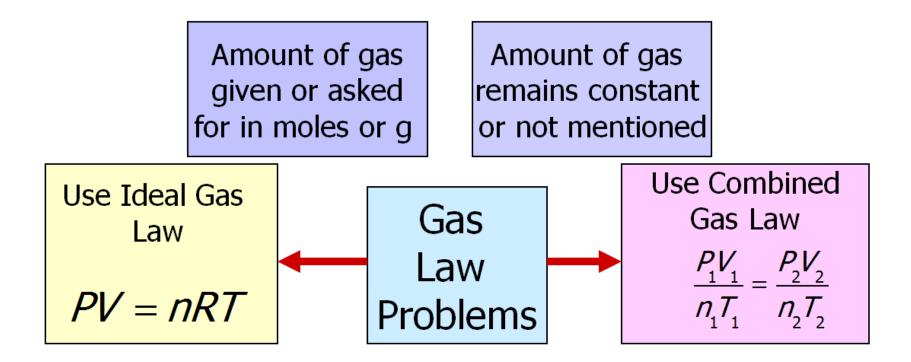
Problem: Calculate the volume occupied by 0.54 mol of N_2 at 15 °C and 0.976 atm.

If given moles, use the ideal-gas law: PV = nRT

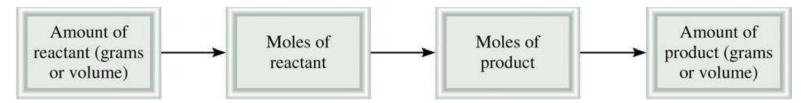




When to use the Ideal Gas Law, or the Combined Gas Law?



Stoichiometry: Stoichiometric connections can be included in gas law type problems:



Problem: What is the pressure in atmospheres in a 35.0 L balloon at 25.0 °C filled with dried hydrogen gas produced by the reaction of 39.8 g of NaH with water?

We need to find the number of moles of hydrogen from the *balanced* equation:

NaH(s) + H₂O(l)
$$\rightarrow$$
 NaOH(aq) + H₂(g)
Molar Mass NaH = 22.99 + 1.01
= 24.00 g/mol
39.8 g NaH x, $\frac{1 \mod \text{NaH}}{24.00 \text{ g NaH}}$ x $\frac{1 \mod \text{H}_2}{1 \mod \text{NaH}}$ = 1.66 mol H₂
Moles of NaH
So put that into: P = $\frac{\text{nRT}}{\text{V}}$ = $\frac{(1.66 \mod)(0.08206 \frac{\text{L atm}}{\text{mol K}})(298 \text{ K})}{(35.0 \text{ L})}$ = 1.16 atm

Further Applications of the Ideal-Gas Equation (to determine molecular weights)

$$\frac{n}{V} = \frac{P}{RT}$$
 if we multiply both sides by the Molecular Weight, M, of the gas:

$$\frac{nM}{V} = \frac{PM}{RT} \qquad \text{but } \frac{nM}{V} = \frac{\text{moles}}{\text{liter}} \cdot \frac{\text{grams}}{\text{mole}} = \frac{\text{grams}}{\text{liter}} = \text{ density (of a gas), d}$$

$$d = \frac{PM}{RT}$$
 rearranged is $M = \frac{dRT}{P}$

Problem: If a 2.00 L flask contains 3.11 g of cyclopropane gas at 684 torr and 23.0 °C, what is the molecular weight of cyclopropane? ($R = 0.08206 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$)

$$M = \frac{dRT}{P} = \frac{\left(\frac{3.11 \text{ g}}{2.00 \text{ L}}\right) (0.08206 \frac{\text{L atm}}{\text{mol K}}) (296 \text{ K})}{\left(684 \text{ torr } \times \frac{1 \text{ atm}}{760 \text{ torr}}\right)} = 42.0 \frac{\text{g}}{\text{mol}}$$

An <u>alternative</u> solution could find the number of *moles*, and then use that to find the MW:

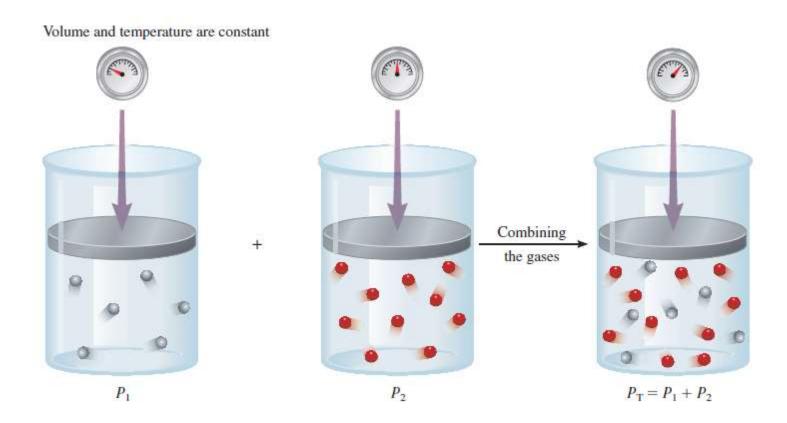
n =
$$\frac{PV}{RT}$$
 = $\frac{\left(684 \text{ torr } \times \frac{1 \text{ atm}}{760 \text{ torr}}\right)}{(0.08206 \frac{L \text{ atm}}{\text{mol K}})} (296 \text{ K})} = 0.0741 \text{ mol}$
23 °C = 296 K

Molar mass =
$$\frac{3.11 \text{ g}}{0.0741 \text{ mol}} = 42.0 \frac{\text{g}}{\text{mol}}$$

Gas Mixtures and Partial Pressures

So far we have talked about pure gaseous substances – but in reality we will encounter mixtures of gases.

Dalton's Law of Partial Pressures (1801) – the total pressure of a mixture of (*non-reacting*) gases equals the sum of the pressures that each would exert if it were present alone.



The total pressure is the same as the *sum* of the grey pressure *and* the red pressure.

We can express this mathematically:

(Assuming each gas behaves *ideally*)

$$P_{total} = P_1 + P_2 + P_3 + \dots$$

$$P_{total} = (n_1 + n_2 + n_3 + \dots) \frac{RT}{V} = n_{total} \left(\frac{RT}{V}\right)$$

$$\frac{P_1}{P_{total}} = \frac{n_1 RT/V}{n_{total} RT/V} = \frac{n_1}{n_{total}}$$
The mole fraction of gas1: $X_1 = \frac{n_1}{n_{total}}$

$$\sum X_i = X_1 + X_2 + X_3 + \dots = 1$$

$$P_1 = \left(\frac{n_1}{n_{total}}\right) P_{total} = X_1 P_{total}$$

 X_i is the mole fraction of i, which is a dimensionless quantity that expresses the *ratio* of the *number* of *moles* of one component to the **total** *number* of *moles* of all the components present.

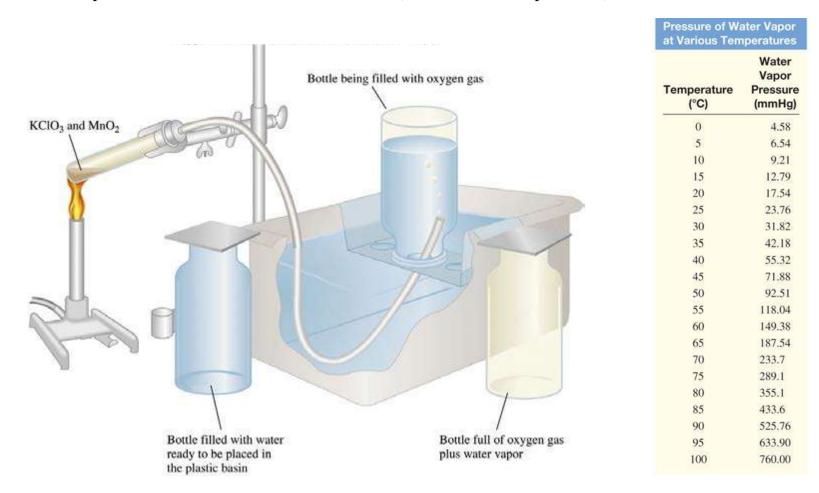
Sometimes mole fraction is also expressed as Mole Percent:

Mole % = $X_a \times 100\%$ $X_a = \frac{1}{2} \implies \text{mole percent} = 50\%.$

This is useful for gases collected over water. E.g. If oxygen is collected over water, then the total pressure is:

$$P_{total} = P_{O_2} + P_{H_2O}$$

The pressure of gas in the bottle is not only due to O_2 , but also to water vapor. The vapor pressure of water varies with temperature. If we are to calibrate the bottle (for amount of O_2 produced), we must take this into account.



Problem: A mixture of gases contains 0.75 mol N_2 , 0.30 mol O_2 , and 0.15 mol CO_2 . If the total pressure is 1.56 atm, what is the partial pressure of each component?

$$X_{N_{2}} = \frac{n_{N_{2}}}{n_{total}} = \frac{0.75 \text{ mol}}{1.20 \text{ mol}} = 0.625$$

$$n_{total} = 0.75 + 0.30 + 0.15$$

$$= 1.20 \text{ mol}$$

$$X_{O_{2}} = \frac{n_{O_{2}}}{n_{total}} = \frac{0.30 \text{ mol}}{1.20 \text{ mol}} = 0.25$$

$$X_{CO_{2}} = \frac{n_{CO_{2}}}{n_{total}} = \frac{0.15 \text{ mol}}{1.20 \text{ mol}} = 0.125$$

$$Check \Sigma = 1$$

$$P_{N_2} = X_{N_2} P_{total} = (0.625) (1.56 \text{ atm}) = 0.98 \text{ atm}$$

$$P_{O_2} = X_{O_2} P_{total} = (0.25) (1.56 \text{ atm}) = 0.39 \text{ atm}$$

$$P_{CO_2} = X_{CO_2}P_{total} = (0.125)(1.56 \text{ atm}) = 0.20 \text{ atm}$$

Problem: At an underwater depth of 250 ft, the pressure is 8.38 atm. What should the mole percent of oxygen be in the diving gas for the partial pressure of oxygen in the mixture to be 0.21 atm, the same as in air at 1 atm?

$$P_{O_2} = X_{O_2} P_{total} \quad \text{rearrange to give} \quad X_{O_2} = \frac{P_{O_2}}{P_{total}}$$
$$= \frac{0.21 \text{ atm}}{8.38 \text{ atm}}$$
$$= 0.025$$

which as a mole percent = 2.5 %

Kinetic-Molecular Theory (Boltzmann and Maxwell – Late 1800's)

The kinetic (*molecular*) theory describes a gas as a large number of *submicroscopic particles* (atoms or molecules), all of which are in constant, rapid motion that have randomness arising from their many collisions with each other and with the walls of the container.

Kinetic theory explains macroscopic properties of gases, such as pressure, temperature, viscosity, thermal conductivity, and volume, by considering their *molecular composition* and *motion*.

The theory posits that gas pressure is due to the impacts on the walls of a container, of molecules or atoms moving at different *velocities*.

Assumptions made:

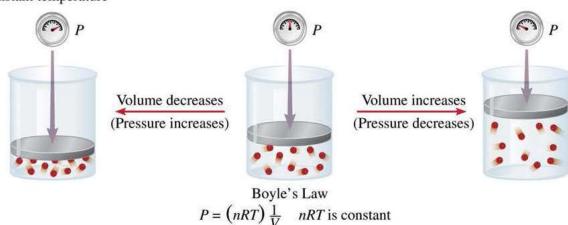
- 1. Gases are collections of molecules in *continuous random motion*.
- 2. Gas molecules are infinitesimally small particles.
- 3. Particles move in straight lines until they collide.
- 4. Molecules do **not** *influence* one another except when they *collide*.

5. The average kinetic energy of the molecules is proportional to the absolute temperature. (At any given temperature the molecules of all gases have the *same* kinetic energy).

Kinetic-Molecular Theory gives us an understanding of pressure and temperature on the molecular level.

- Pressure results from *collisions* of molecules with the *walls* of the container.
- The magnitude of pressure is determined by how *often* and how *hard* the molecules strike the walls of the container.
- Absolute temperature of a gas is a measure of the *average kinetic energy* of the molecules.
- Some molecules will have *less* kinetic energy and some will have *more* kinetic energy than the average (distribution).

Visual Illustrations

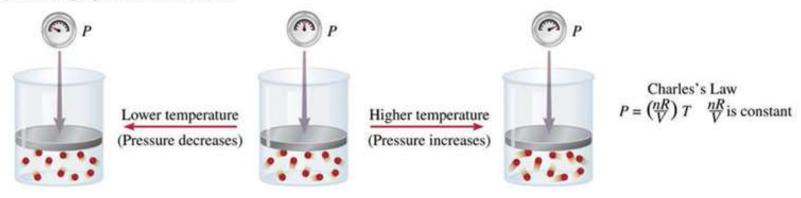


Increasing or decreasing the volume of a gas at a constant temperature

More wall collisions = Higher P

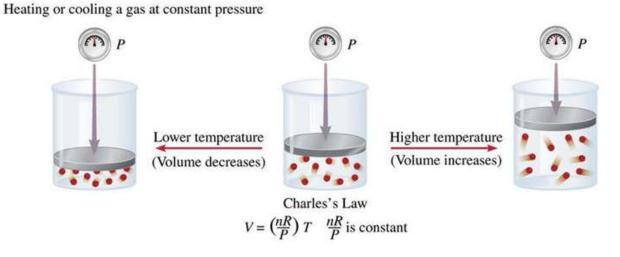
Less wall collisions = Lower P

Heating or cooling a gas at constant volume



Lower T = *slower motion* (*softer collisions*)= *Lower P*

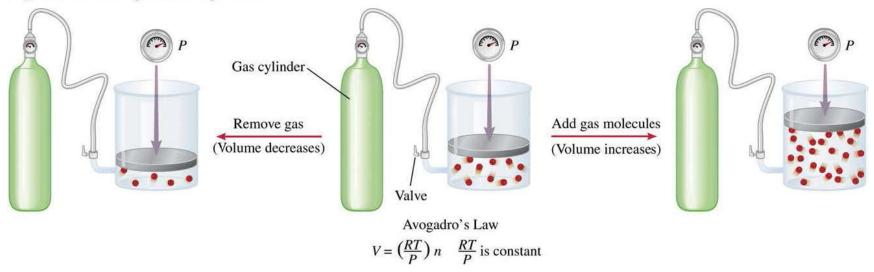
Higher T = *more ke* = *harder collisions*= *Higher P*



Lower T = slower motion, need smaller V to maintain P.

Higher T = more motion, need larger V to keep P

Dependence of volume on amount of gas at constant temperature and pressure



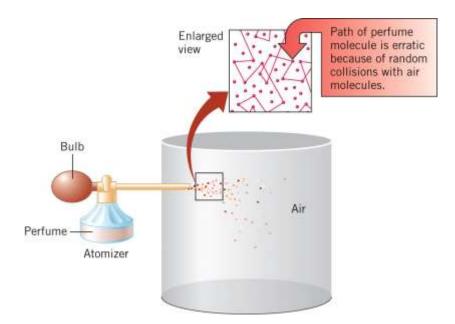
Less molecules means need smaller V to maintain P.

More molecules means need bigger V to keep P.

We now are able to address gaseous motion, and there are two different types: diffusion and effusion...

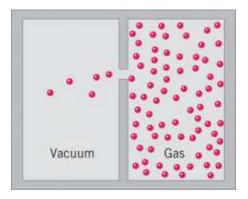
Diffusion is the *gradual mixing* of molecules of one gas with molecules of another, by virtue of their *kinetic properties*.

(E.g. smelling your food).



Effusion is the process by which a gas under pressure escapes from one compartment of a container to another by passing through a *small opening*.

Effusion is the process in which a gas escapes through a hole of diameter *considerably smaller* than the mean free path of the molecules.



(The mean free path is the average distance traveled by a gas molecule between collisions. At sea level, the mean free path for air molecules is about 6×10^{-6} cm).

Graham's Law of Effusion (1832)

Graham studied the rates of effusion (and diffusion) of gases, and their relationship to molecular weight.

He found that the rate of effusion is inversely proportional to the (square root of the) molecular weight of the gas.

Rate of effusion
$$\propto \frac{1}{\sqrt{molar mass}}$$
 Of Rate of effusion $\propto \frac{1}{\sqrt{M}}$
Average speed $\propto \frac{1}{\sqrt{molar mass}}$

Graham's Law is usually used to compare the rates of effusion of two different gases:

For two gasses **A** and **B** (at the same Temperature and Pressure):

$$\frac{\text{rate of effusion of A}}{\text{rate of effusion of B}} = \frac{1/\sqrt{M_A}}{1/\sqrt{M_B}} = \sqrt{\frac{M_B}{M_A}} \qquad \qquad \frac{\text{Time for A to effuse}}{\text{Time for B to effuse}} = \sqrt{\frac{M_A}{M_B}}$$

$$\frac{\text{rate of effusion of A}}{\text{rate of effusion of B}} = \frac{\text{time for B to effuse}}{\text{time for A to effuse}} = \sqrt{\frac{M_B}{M_A}}$$

The rate of effusion is the *number of molecules passing through a point in a given time*. The longer the time it takes, the slower is the rate. Therefore the rate is *inversely proportional* to the time required for effusion.

Distribution of Molecular Speeds

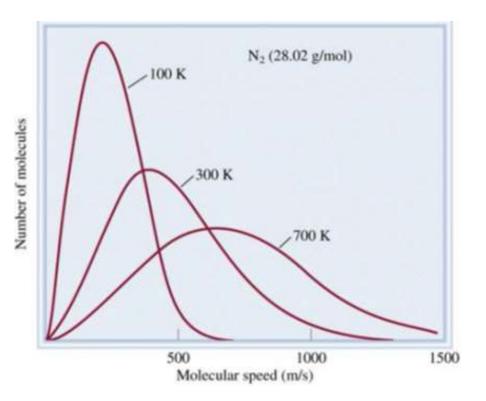
We previously said the absolute temperature of a gas is a measure of the average kinetic energy of the molecules, which is related to the speed of the molecules ($\mathbf{ke} = \frac{1}{2}\mathbf{mv}^2$).

This Maxwell speed distribution curve is for N_2 at three *different temperatures*, displaying the **number** of molecules moving at a certain **speed**.

The speed at the top of the curve is called the most *probable* speed because the largest number of molecules have that speed.

At lower temperatures, the molecules have less energy. Therefore, the speeds of the molecules are lower and the distribution has a smaller range.

As the temperature of the molecules increases, the distribution flattens out. Since the molecules have *greater energy* at higher temperature, the molecules are moving *faster*.

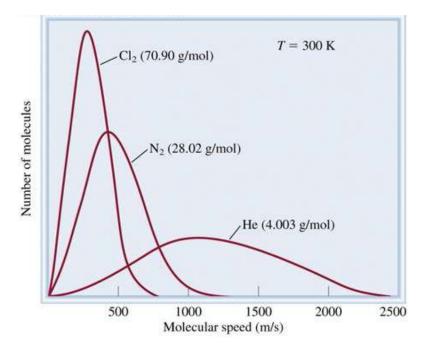


This graph shows the **speed** distribution of three *different* gases at the <u>same</u> temperature.

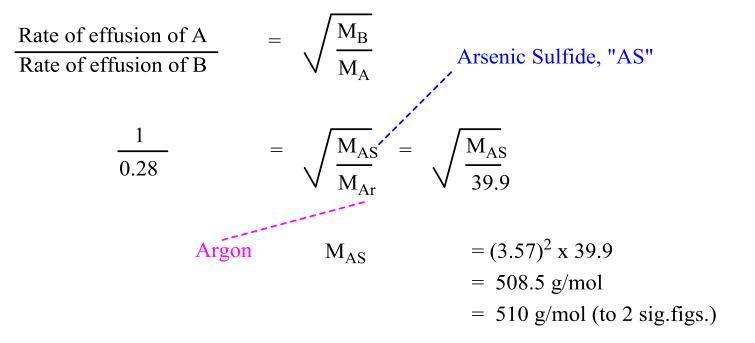
On average, heavier molecules move more slowly than lighter molecules.

(Lighter molecules move faster, on average, than heavier molecules).

Therefore, *heavier* molecules will have a *smaller* speed *distribution*, while **lighter** molecules will have a speed **distribution** that is more **spread out**.



Problem: Arsenic (III) sulfide sublimes readily. The molecules of the vapor phase are found to effuse through a tiny hole at 0.28 times the rate of effusion of Ar atoms under the same conditions of temperature and pressure. What is the molecular formula of arsenic (III) sulfide in the gas phase?



Since As = 74.9 and S = 32.1, $As_2S_3 = 246.1$ g/mol

so it looks like in the gas phase it is *dimeric*, As_4S_6

Real Gases: Deviations from Ideal Behavior

Assumptions for Ideal Behavior included:

- Gas molecules do not exert forces, attractive or repulsive, on each other.
- The volume of the molecules is negligibly small compared with the volume of the container.

Ideal behavior is seen at lower pressures and higher temperatures.

But if we move away from these conditions, we observe that gases exhibit more non-ideal behavior at higher pressures and lower temperatures.

The van der Waals equation (or van der Waals equation of state, 1873) can be viewed as an adjustment to the ideal gas law that takes into account the **non-zero volume** of gas molecules, which are subject to an **inter-particle attraction**.

We know ideally that **PV** = **nRT**, but we can correct for molecular attraction, and volume of molecules:

$$\left(P+\frac{n^2a}{V^2}\right)\left(V-nb\right)=nRT$$

a – magnitude of a reflects how strongly the gas molecules attract each other; has units of $L^2 \cdot atm/mol^2$ b – a measure of actual intrinsic volume occupied by a mole of gas molecules; has units of L/mol.